Types, order, and degree of differential equations

Indicate the type, order, and degree of each of the following differential equations:

1. $y' = 2x$
2. $xdy - ydx = 0$
3. $y'' + {y'}^2 + y = 0$
4. ${y'}^2 = 4 - y^2$
5. $y'' + 5x + y = 0$
6. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y}$
7. $y''' - 4y' = 3e^{2x}$
8. $y''' - 6y'' + 2y' + 36y = 0$
9. $x^2 y'' - 2xy' - 4y = 0$
10. $x^2 y'' + xy' + y = \ln x$

Solutions

1. $$y' = 2x$$

   We can indicate the type, order, and degree as follows:

   - **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to a single independent variable.
   - **Order**: First order, because the highest order derivative is of the first order (y').
   - **Degree**: First degree, because the highest order derivative appears with an exponent of 1 and is not raised to any power or within a non-algebraic function.

2. $$x\,dy - y\,dx = 0$$

   Type, order, and degree of this differential equation:

   - **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to a single independent variable.
   - **Order**: First order, because the highest order derivative is of the first order ($\frac{dy}{dx}$).
   - **Degree**: First degree, because the highest order derivative appears with an exponent of 1 and is not raised to any power or within a non-algebraic function.

3. $$y'' + (y')^2 + y = 0$$

   Type, order, and degree of this differential equation:

   - **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to a single independent variable.
   - **Order**: Second order, because the highest order derivative is of the second order (y'').
   - **Degree**: First degree, because the highest order derivative (y'') appears with an exponent of 1 and is not raised to any power or within a non-algebraic function.

4. $$(y')^2 = 4 - y^2$$

   Type, order, and degree of this differential equation:

   - **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to a single independent variable.
   - **Order**: First order, because the highest order derivative is of the first order (y').
   - **Degree**: Second degree, because the highest order derivative (y') is squared.

5. $$y'' + 5x + y = 0$$

   Type, order, and degree of this differential equation:

   - **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to a single independent variable.
   - **Order**: Second order, because the highest order derivative is of the second order (y'').
   - **Degree**: First degree, because the highest order derivative (y'') appears with an exponent of 1 and is not raised to any power or within a non-algebraic function.

6.

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y}$$

Type, order, and degree of this differential equation:

- **Type**: Partial differential equation (PDE), as it involves partial derivatives with respect to more than one independent variable.
- **Order**: Second order, because the highest order derivative is of the second order $\left(\frac{\partial^2 z}{\partial x^2}\right)$.
- **Degree**: First degree, because the highest order derivative appears with an exponent of 1 and is not raised to any power or within a non-algebraic function.

7.

$$y''' - 4y' = 3e^{2x}$$

- **Type**: Ordinary differential equation (ODE), because it involves ordinary derivatives of y with respect to the independent variable x.
- **Order**: Third order, as the highest order derivative present is y'''.
- **Linearity**: Linear, because y and its derivatives appear linearly (not raised to powers or multiplied by each other).
- **Homogeneity**: **Non-homogeneous**, because there is a non-zero term $3e^{2x}$ on the right side of the equation.
- **Coefficients**: Constant, as the coefficients of y''' and y' are constant numbers (1 and -4, respectively).

8.

$$y''' - 6y'' + 2y' + 36y = 0$$

- **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives of y with respect to x.
- **Order**: Third order, because the highest order derivative is y'''.
- **Linearity**: Linear, because y and its derivatives appear to the first degree and are not multiplied by each other.
- **Homogeneity**: **Homogeneous**, because the independent term is zero; all terms contain y or its derivatives.
- **Coefficients**: Constant, as the coefficients (-6, 2, and 36) are constant numbers.

9.

$$x^2y'' - 2xy' - 4y = 0$$

- **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives with respect to x.
- **Order**: Second order, as the highest order derivative is y''.
- **Linearity**: Linear, because y and its derivatives appear linearly and are not raised to powers or multiplied by each other.
- **Homogeneity**: **Homogeneous**, because the independent term is zero; all terms depend on y or its derivatives.
- **Coefficients**: Variable, as the coefficients x^2 and $-2x$ depend on the independent variable x.

10.

$$x^2y'' + xy' + y = \ln x$$

- **Type**: Ordinary differential equation (ODE), as it involves ordinary derivatives of y with respect to x.
- **Order**: Second order, as the highest order derivative present is y''.
- **Linearity**: Linear, because y and its derivatives appear in the first degree and are not multiplied by each other.
- **Homogeneity**: **Non-homogeneous**, because there is a non-zero term $\ln x$ on the right side.
- **Coefficients**: Variable, as the coefficients x^2 and x are functions of the independent variable x.